SOLUTIONS OF PROBLEMS.

2669. Proposed by S. A. COREY, Albia, Iowa.

Let $A_1, A_2, \cdots, A_8$, and $-(A_1 + A_2 + \cdots + A_8)$ be the vector sides of an enneagon, plane or gauche. Also let $B_1, B_2, \cdots, B_8$, and $-(B_1 + B_2 + \cdots + B_8)$ be the vector sides of a second enneagon, where $B_1 = C_1A_1 - C_2C_5A_3 - C_3C_6A_5 + C_4C_5C_6A_7$, $B_2 = C_1A_2 - C_2C_5A_4 - C_3C_6A_6 + C_4C_5C_6A_8$, $B_3 = C_2A_1 + C_1A_3 - C_4C_6A_5 - C_3C_6A_7$, $B_4 = C_2A_2 + C_1A_4 - C_4C_6A_6 - C_3C_6A_8$, $B_5 = C_3A_1 + C_4C_5A_3 + C_1A_5 + C_2C_5A_7$, $B_6 = C_3A_2 + C_4C_5A_4 + C_1A_6 + C_2C_5A_8$, $B_7 = C_4A_1 - C_3A_3 + C_2A_5 - C_1A_7$, and $B_8 = C_4A_2 - C_3A_4 + C_2A_6 - C_1A_8$, C_1, C_2, C_3, C_4, C_5, and C_6 being scalars.

Then, if a_s = tensor A_s, b_s = tensor B_s, and $\cos(A_rA_s)$ = cosine of the angle included between A_r and A_s and $\cos(B_rB_s)$ = cosine of the angle included between B_r and B_s, establish the following relation between the sides and angles of the two enneagons:

$$[C_1^2 + C_5C_2^2 + C_6C_3^2 + C_5C_6C_4^2][a_1a_2\cos(A_1A_2) + C_5a_3a_4\cos(A_3A_4) + C_6a_5a_6\cos(A_5A_6) + C_5C_6a_7a_8\cos(A_7A_8)]$$
$$= b_1b_2\cos(B_1B_2) + C_5b_3b_4\cos(B_3B_4) + C_6b_5b_6\cos(B_5B_6) + C_5C_6b_7b_8\cos(B_7B_8).$$

Show that Geometry problem 506 is a special case of the foregoing. Give illustrative example, using triangle or other simple geometric figure, by assuming that some of the sides of the first enneagon are zero.

SOLUTION BY PROPOSER.

Whenever $A_1, A_2, \cdots$ and A_8 are scalar (or algebraic) quantities, and $B_1, B_2, \cdots$ and B_8 have scalar values corresponding in form to those given in the problem, we have the algebraic identity,

$$(C_1^2 + C_5C_2^2 + C_6C_3^2 + C_5C_6C_4^2)(A_1A_2 + C_5A_3A_4 + C_6A_5A_6 + C_5C_6A_7A_8)$$
$$= B_1B_2 + C_5B_3B_4 + C_6B_5B_6 + C_5C_6B_7B_8.$$

Inasmuch as all the terms in $A_1, A_2, \cdots A_8$, and $B_1, B_2, \cdots B_8$ in this algebraic identity are of the second degree, a geometric interpretation may be obtained by assuming that $A_1, A_2, \cdots A_8$ and $B_1, B_2, \cdots B_8$, are vectors. This follows immediately from the fact that vector multiplication is commutative in so far as the scalar part of the product is concerned whenever all the vector terms employed are of the second degree. But the scalar part of the vector product B_rB_s is $-b_rb_s\cos(B_rB_s)$. Substituting this scalar part of the vector product in both members of the above algebraic equation and changing signs we obtain at once the equation contained in the problem. If $A_1 = A_2$, $A_3 = A_4$, $A_5 = A_6$, and $A_7 = A_8$, the problem becomes identical with Geometry problem 506.

Example. As long as a vector maintains a constant length and direction in space, its origin in space may be altered at will. Hence we need not confine our attention to closed geometric figures in interpreting the given identity. Let DEF be a given triangle. Bisect DE in G. Draw GF and extend EF to F'. Draw GH intersecting DF in I and EF' in J. Let $A_1 = GF$, $A_2 = GI$, and $A_3 = GD$. Also let $A_4 = A_5 = A_6 = A_7 = A_8 = 0$, $C_1 = C_2 = C_3 = C_4 = C_5 = 1$, and $C_6 = 0$. Substituting in the given identity, paying strict attention to the direction of the vectors employed, and dividing by the constant factor, we readily get $2GF\cos FGI = DF\cos GID + EF\cos EJG$, a known result.

2670. Proposed by CLIFFORD N. MILLS, Brookings, South Dakota.

A telegraph wire, weighing one tenth pound per yard, is stretched between poles on level ground, so that the greatest dip of the wire is three feet. Find approximately the distance between the poles when the tension at the lowest point of the wire is 140 pounds.

SOLUTION BY ELBERT H. CLARKE, Hiram College, Ohio.

It is a well-known property of wires hanging freely from two supports that the tension at